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20. Abstract: A Mathematical model of the three-dimensional two-phase reacting flows in gas turbine combustors has been developed which takes into account the mass, momentum and energy coupling between the phases. The fundamental equations of motion of the droplets are solved numerically in a Lagrangian frame of reference using a finite difference solution of the governing equations of the gas. Well known relations are used to model the heat and mass transfer processes and the initial droplet heat-up is allowed for. The entire fuel spray is constructed using a finite number of size ranges obeying a two parameter droplet size distribution. The results are found to be in close agreement with experimental data. An important feature of this analytical technique is that it permits the rational selection or specification of fuel nozzle design.

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Abstract

A mathematical model of the three-dimensional two-phase reacting flows in gas turbine combustors has been developed which takes into account the mass, momentum and energy coupling between the phases. The fundamental equations of motion of the droplets are solved numerically in a Lagrangian frame of reference using a finite difference solution of the governing equations of the gas. Well known relations are used to model the heat and mass transfer processes and the initial droplet heat-up is allowed for. The entire fuel spray is constructed using a finite number of size ranges obeying a two parameter droplet size distribution. The results are found to be in close agreement with experimental data. An important feature of this analytical technique is that it permits the rational selection or specification of fuel nozzle design.

Introduction

The advent of more powerful digital computers has provided the means whereby mathematical modelling can be applied to gas turbine combustor problems to facilitate greatly the arduous initial stages of combustor design. This is now of particular interest in view of the current demands which combustion designers are required to meet in particular efficiency of combustion at a wide range of operating conditions and the strict control of pollutant emissions. The latter have become increasingly stringent in recent years for economic and political reasons. The present trend is away from the traditional cut-and-try methods, which are expensive and do not necessarily produce the optimum answer, towards fundamental modelling of the physical and chemical processes occurring within the combustor.

In this study, the problem of three-dimensional two-phase combustion is approached with the aim of producing an algorithm based on fundamental principles which can correlate all the details of combustion occurring within a gas turbine combustor can. It has been shown by Mellor[1] that a key factor in gas turbine combustion is the effective fuel distribution which is determined by the fuel preparation technique. It has also been indicated by Faeth and co-workers[2], that the behaviour of a liquid spray flame approaches that of a gas diffusion flame provided a sufficiently fine spray is considered. It is therefore essential that the model incorporates an accurate representation of the droplet size distributions encountered in gas turbine combustors, and solves the relevant equations for trajectory and evaporation taking into account the heat, mass and momentum interaction between the phases.

In liquid-fuelled gas turbine combustors the liquid disintegrates and atomizes soon after leaving the injector forming a spectrum of droplet sizes. The droplets evaporate along the trajectories that they follow leaving behind pockets of fuel vapour which later mix with the oxidiser and burn. Although a large body of literature exists on the combustion of single drops of fuel[3] it has been recently indicated by Chigier[4] that, in the light of recent experimental research, the validity of this information for spray combustion is questionable. There is evidence that the spray acts as a permeable surface of fuel vapour the properties of which depend on the interaction between the gas flow and the individual droplets.

Previous studies by Swithenbank and co-workers[5] on modelling combustors involved the adaptation of a finite-domain procedure for the computation of gas flows developed at Imperial College[6-9]. In order to model the liquid spray several size ranges were each treated as a chemical species and defined in terms of an elliptic equation in the solution procedure. The trajectories of the individual droplets were analysed in two dimensions by stipulating that flow in the vicinity of the atomiser is axially symmetric in a supplementary study[10].

In order to more realistically model the complex interactions between the two phases a new strategy has now been adopted in which the gas phase is calculated in an Eulerian frame of reference whilst a separate calculation is performed for the liquid phase in a Lagrangian frame. The latter is achieved by applying the Particle-Source-In-Cell method[11] to the three-dimensional geometry of the present problem. This approach has several advantages over the previous method and it is believed that ultimately this will enable the designer to evaluate a particular combustor design without the need to build a prototype, and to give him more detailed information on performance than is at present available.

In particular, this technique reveals details of the three dimensional spray trajectories and the consequent evaporated fuel distribution, thus permitting the rational selection of fuel nozzle characteristics. It can therefore fill an important gap in current combustor design procedure.

Equations to be solved

Since most gas turbine combustors exhibit a degree of cylindrical cyclic symmetry in their design, it is convenient to present the governing equations in cylindrical co-ordinates.

The mean motion of the gas is determined by the well-established differential equations of the conservation of mass and momentum.

Mass conservation

$$\frac{1}{r}\frac{\partial}{\partial r}(\rho r v) + \frac{1}{r}\frac{\partial}{\partial \theta}(\rho w) + \frac{\partial}{\partial x}(\rho u) = \dot{m}'' \quad \ldots\ldots (1)$$

x-momentum

$$\frac{1}{r}\left[\frac{\partial p}{\partial x}(r\rho u^2) + \frac{\partial}{\partial r}(r\rho v u) + \frac{\partial}{\partial \theta}(\rho w u)\right] = -\frac{\partial p}{\partial x} + \frac{1}{r}\frac{\partial}{\partial r}$$

$$\left[r\mu\left\{\frac{\partial u}{\partial r} + \frac{\partial v}{\partial x}\right\}\right] + \frac{1}{r}\frac{\partial}{\partial \theta}\left[\mu\left\{\frac{1}{r}\frac{\partial u}{\partial \theta} + \frac{\partial w}{\partial x}\right\}\right] + \frac{\partial}{\partial x}\left[2\mu\frac{\partial u}{\partial x}\right] + F_x \qquad \ldots\ldots (2)$$

y-momentum

$$\frac{1}{r}\left[\frac{\partial}{\partial x}(r\rho u v) + \frac{\partial}{\partial r}(r\rho v^2) + \frac{\partial}{\partial \theta}(\rho w v) - \rho w^2\right] = -\frac{\partial p}{\partial r} + \frac{1}{r}\frac{\partial}{\partial r}$$

$$\left[2r\mu\frac{\partial v}{\partial r}\right] + \frac{1}{r}\frac{\partial}{\partial \theta}\left[\mu\left\{r\frac{\partial}{\partial r}\left(\frac{w}{r}\right) + \frac{1}{r}\frac{\partial v}{\partial \theta}\right\}\right] - 2\frac{\mu}{r}\left[\frac{1}{r}\frac{\partial w}{\partial \theta} + \frac{v}{r}\right]$$

$$+ \frac{\partial}{\partial x}\left[\mu\left(\frac{\partial u}{\partial r} + \frac{\partial v}{\partial x}\right)\right] + F_r \qquad \ldots\ldots (3)$$

θ-momentum

$$\frac{1}{r}\left[\frac{\partial}{\partial x}(r\rho u w) + \frac{\partial}{\partial r}(r\rho v w) + \frac{\partial}{\partial \theta}(\rho w^2) + \rho v w\right] = -\frac{1}{r}\frac{\partial p}{\partial \theta} + \frac{1}{r}\frac{\partial}{\partial \theta}$$

$$\left[r\mu\left\{r\frac{\partial}{\partial r}\left(\frac{w}{r}\right) + \frac{1}{r}\frac{\partial v}{\partial \theta}\right\}\right] + \frac{\mu}{r}\left[r\frac{\partial}{\partial r}\left(\frac{w}{r}\right) + \frac{1}{r}\frac{\partial v}{\partial \theta}\right] + \frac{1}{r}\frac{\partial}{\partial \theta}$$

$$\left[\mu\left\{\frac{2}{r}\frac{\partial w}{\partial \theta} + \frac{2v}{r}\right\}\right] + \frac{\partial}{\partial x}\left[\mu\left(\frac{\partial w}{\partial x} + \frac{1}{r}\frac{\partial u}{\partial \theta}\right)\right] + F_\theta \qquad \ldots\ldots (4)$$

In these equations, u, v and w are the velocity components in the x,r and θ directions; p is the pressure, ρ and μ are the density and the viscosity. It can be seen that the mass conservation equation contains the source term $\dot{m}'''$ which is due to the mass addition from the spray to the gas by evaporation, and the momentum interaction between the two phases is contained in the terms, F_x, F_r and F_θ. The usual Reynolds stresses have been replaced in the above equations by the effective viscosity hypothesis, where μ represents the effective viscosity (i.e. the molecular viscosity augmented by its 'turbulent' counterpart).

The distribution of the turbulent viscosity is provided by the k-ε turbulence model used. It entails the solution of two transport equations for turbulence characteristics, namely the kinetic energy of the fluctuating motion and its dissipation rate. The differential transport equations for k and ε are

$$\frac{1}{r}\left[\frac{\partial}{\partial x}(r\rho u k) + \frac{\partial}{\partial r}(r\rho v k) + \frac{\partial}{\partial \theta}(\rho w k)\right] = \frac{1}{r}\frac{\partial}{\partial r}\left[r\frac{\mu}{\sigma_k}\frac{\partial k}{\partial r}\right] +$$

$$\frac{1}{r}\frac{\partial}{\partial \theta}\left[\frac{\mu}{r_k}\frac{\partial k}{\partial \theta}\right] + \frac{\partial}{\partial x}\left[\frac{\mu}{\sigma_k}\frac{\partial k}{\partial x}\right] + G_k - \rho\varepsilon \qquad \ldots\ldots (5)$$

and

$$\frac{1}{r}\left[\frac{\partial}{\partial x}(r\rho u \varepsilon) + \frac{\partial}{\partial r}(r\rho v \varepsilon) + \frac{\partial}{\partial \theta}(\rho w \varepsilon)\right] = \frac{1}{r}\frac{\partial}{\partial r}\left[r\frac{\mu}{\sigma_\varepsilon}\frac{\partial \varepsilon}{\partial r}\right] +$$

$$\frac{1}{r}\frac{\partial}{\partial \theta}\left[\frac{\mu}{\sigma_\varepsilon}\frac{1}{r}\frac{\partial \varepsilon}{\partial \theta}\right] + \frac{\partial}{\partial x}\left[\frac{\mu}{\sigma_\varepsilon}\frac{\partial \varepsilon}{\partial x}\right] + C_1\frac{\varepsilon}{k}G_k - C_2\rho\frac{\varepsilon^2}{k}$$

$$\ldots\ldots (6)$$

where G_k is the generation term of k and is given by

$$G_k = \mu\left[2\left\{\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial r}\right)^2 + \left(\frac{1}{r}\frac{\partial w}{\partial \theta} + \frac{v}{r}\right)^2\right\} + \left(\frac{\partial u}{\partial r} + \frac{\partial v}{\partial x}\right)^2 +\right.$$

$$\left. + \left(\frac{\partial w}{\partial x} + \frac{1}{r}\frac{\partial u}{\partial \theta}\right)^2 + \left(\frac{1}{r}\frac{\partial v}{\partial \theta} + \frac{\partial w}{\partial r} - \frac{w}{r}\right)^2\right] \qquad \ldots\ldots (7)$$

The turbulent viscosity is related to k and ε by dimensional arguments in the following way:

$$\mu_t = C_\mu \rho \frac{k^2}{\varepsilon} \qquad \ldots\ldots (8)$$

The coefficients C_1, C_2, C_μ, σ_k and σ_ε are constants which are assigned the following valves:
$C_1 = 1.44$, $C_2 = 1.92$, $C_\mu = 0.09$, $\sigma_k = 1.0$ and $\sigma_\varepsilon = 1.3$.

The additional conservation equations required for a chemically reacting system are those for enthalpy and chemical species. The equation for stagnation enthalpy h can be written as

$$\frac{1}{r}\left[\frac{\partial}{\partial x}(r\rho u h) + \frac{\partial}{\partial r}(r\rho v h) + \frac{\partial}{\partial \theta}(\rho w h)\right] = \frac{1}{r}\frac{\partial}{\partial r}\left[r\frac{\mu}{\sigma_h}\frac{\partial h}{\partial r}\right] +$$

$$\frac{1}{r}\frac{\partial}{\partial \theta}\left[\frac{1}{r}\frac{\mu}{\sigma_h}\frac{\partial h}{\partial \theta}\right] + \frac{\partial}{\partial x}\left[\frac{\mu}{\sigma_h}\frac{\partial h}{\partial x}\right] + S_h \qquad \ldots\ldots (9)$$

where S_h includes radiation sources, shear work and spray-gas interaction terms, and σ_h is the turbulent Prandtl number for enthalpy. The definition of stagnation enthalpy is

$$h = C_p T + m_{fu} H + \frac{1}{2}(u^2 + v^2 + w^2) \qquad \ldots\ldots (10)$$

which is used for the recovery of temperature. In this equation m_{fu} is the mass fraction of unburned fuel in the mixture, H is its heat of reaction, C_p is the specific heat of the mixture at constant pressure and T is temperature.

The conservation equation for a chemical species j, takes the following form

$$\frac{1}{r}\left[\frac{\partial}{\partial x}(r\rho u m_j) + \frac{\partial}{\partial r}(r\rho v m_j) + \frac{\partial}{\partial \theta}(\rho w m_j)\right] = \frac{1}{r}\frac{\partial}{\partial r}\left[r\frac{\mu_t}{\sigma_j}\frac{\partial m}{\partial r}\right] +$$

$$\frac{1}{r}\frac{\partial}{\partial \theta}\left[\frac{1}{r}\frac{\mu}{\sigma_j}\frac{\partial m_j}{\partial \theta}\right] + \frac{\partial}{\partial x}\left[\frac{\mu}{\sigma_j}\frac{\partial m_j}{\partial x}\right] + R_j + S_j \qquad \ldots\ldots (11)$$

where, m_j is the mass fraction of species k. R_j is the mass rate of creation or depletion by chemical reaction and S_j is the rate of creation by evaporation from the liquid phase. If it is assumed that the combustion reaction is a single step kinetically influenced reaction between a pair of reactants to form a single product, it is necessary to solve one such equation for each of m_{fu} and m_{ox}. The influence of turbulence on the rate of reaction is taken into account by employing the eddy break up model of Spalding[12]. The reaction rate for m_{fu} is in this case taken to be the smaller of two expressions given by the familiar Arrhenius formulation and the eddy break up model. The latter is conveniently described as

$$R_{fu,EBU} = -C_R \rho k^{\frac{1}{2}}\left[\left(\frac{\partial m_{fu}}{\partial x}\right)^2 + \left(\frac{\partial m_{fu}}{\partial r}\right)^2 + \left(\frac{1}{r}\frac{\partial m_{fu}}{\partial \theta}\right)^2\right]^{\frac{1}{2}}$$

$$\ldots\ldots (12)$$

Where, C_R is a constant of order 6.

The density distribution in the flow is provided by the equation of state;

$$\rho = P/RT \sum_j \frac{m_j}{M_j} \qquad \ldots\ldots(13)$$

Here M_j is the molecular weight of species j and R is the gas constant.

The effects of radiation are accounted for in the present study by reference to the fluxes in the positive and negative x, r and θ directions[13,14] The six first order differential equations for these fluxes can be transformed into a set of three second order equations for the composite fluxes R^x, P^r and R^θ, which are expressed as,

$$\frac{d}{dx}\left[\left(\frac{1}{a+s}\right)\frac{dR^x}{dx}\right] = a\,(R^x - E) + \frac{s}{3}\,(2R^x - R^r - R^\theta) \qquad \ldots\ldots(14)$$

$$\frac{d}{dr}\left[\left(\frac{r}{a+s+\frac{1}{r}}\right)\frac{dR^r}{dr}\right] = r\left[a(R^r - E) + \frac{s}{3}\,(2R^r - R^x - R^\theta\right] \qquad \ldots(15)$$

$$\frac{d}{d\theta}\left[\left(\frac{1}{a+s}\right)\frac{1}{r}\frac{dR^\theta}{d\theta}\right] = a(R^\theta - E) + \frac{s}{3}\left[2R^\theta - R^x - R^r\right] \qquad \ldots(16)$$

where, a is the flux absorption, coefficient, s is the scattering coefficient, $E=\sigma T^4$ is the black body emissive power at the fluid temperature and σ is the Stefan-Boltzmann constant. The contribution of radiation to the source term of the enthalpy equation is calculated from

$$S_{h,rad} = 2a[R^x + R^r + R^\theta - 3E] \qquad \ldots\ldots(17)$$

Those source terms of the above equations (1), (2), (3), (4), (9) and (11) which arise from the interaction between the gas and the droplets are obtained from the solution of the equations of the spray. Formulations of spray motion in both Eulerian[15] and Lagrangian[16] frames have been reported. The latter method is more suitable for flows where the spray is initially confined to a thin sheet[17] and has therefore been adopted in the present study.

The initial size distribution of the spray is divided into an adequate number of discrete intervals each represented by an average diameter. If it is assumed that the size distribution is of the Rosin-Rammler type, the mass fraction of drops of diameter greater than D is given by

$$M_D = \exp\left[-(D/\bar{D})^n\right] \qquad \ldots\ldots(18)$$

where, $\bar{D}$ is the size constant and n is the size distribution parameter. It should be emphasized that the use of the Rosin-Rammler distribution is not necessary for the calculation, however our experiments have shown that the characteristics of most nozzles can be presented in this compact and convenient form.

When all external effects except the drag force are neglected[18], the equations of the motion of each of the droplets representing the size groups which constitute the spray can be written as:

$$\frac{du_p}{dt} = -\left(\frac{18\mu_g}{\rho D^2}\right)\left(\frac{C_D Re}{24}\right)(u_p - u) \qquad \ldots\ldots(19)$$

$$\frac{dv_p}{dt} = \frac{w_p^2}{r_p} - \left(\frac{18\mu_g}{\rho_\ell D^2}\right)\left(\frac{C_D Re}{24}\right)(v_p - v) \qquad \ldots\ldots(20)$$

$$\frac{dw_p}{dt} = -\frac{v_p w_p}{r_p} - \left(\frac{18\mu_g}{\rho_\ell D^2}\right)\left(\frac{C_D Re}{24}\right)(w_p - w) \qquad \ldots\ldots(21)$$

and the equations of trajectory are

$$\frac{dx}{dt} = u_p,\ \frac{dr}{dt} = v_p \text{ and } r\frac{d\theta}{dt} = w_p \qquad \ldots\ldots(22).(23).(24$$

where, μ_g is the molecular viscosity of the gas, ρ_ℓ is the liquid density, C_D is the drag coefficient and u_p, v_p and w_p are the components of the gas velocity in the x, r and θ directions respectively. Re stands here for the relative Reynolds number, which is defined as,

$$Re = \rho_g D|\vec{u}_p - \vec{u}|/\mu_g \qquad \ldots\ldots(25)$$

The Reynolds number dependence of the draf coefficient in sprays is given by the following relations[19]

$$C_D = 27\ Re^{-0.84} \qquad 0 < Re < 80$$
$$C_D = 0.271\ Re^{0.271} \qquad 80 < Re < 10^4 \qquad \ldots\ldots(26)$$

For evaporating droplets an additional equation which gives the rate of change of diameter with respect to time is also needed. This can be expresse as:

$$\frac{dD}{dt} = -\left(\frac{C_b}{2D}\right)(1 + 0.23\ Re^{\frac{1}{2}}) \qquad \ldots\ldots(27)$$

where the second bracketed term on the right hand side accounts for the forced convective augmentation of evaporation and C_b is the vaporization rate constant whose value depends on the physical properties of the surrounding medium as well as the fuel itself. A widely quoted expression for C_b, derived from the quasi-steady analysis of droplet combustion is due to Wise and Agoston[20].

$$C_b = \frac{8\lambda}{\rho_\ell C_p}\,\ell n\left[1 + \frac{C_p}{L}(T_\infty - T_L)\right] \qquad \ldots\ldots(28)$$

Where, λ and C_p are the thermal conductivity and the specific heat at constant pressure of the surrounding gas, L is the latent heat of evaporation, and T_∞ and T_L are the gas and droplet temperatures respectively.

To allow for the 'heating-up' time of the droplets after entry into the gas stream, it is assumed that evaporation begins when the drop temperature reaches the boiling point. The equation for the time rate of change of drop temperature can be expressed as

$$\frac{dT}{dt} = 6\lambda(2+0.3\ Re^{\frac{1}{2}}Pr^{\frac{1}{3}})\ \frac{(T_\infty - T_L)}{(\rho_L D^2 C_{p,L})} \qquad \ldots\ldots(29)$$

where the Nusselt number has been replaced by the familiar Ranz and Marshall[21] correlation.

The location, velocity, size and temperature of each size range of droplets are determined as

The location, velocity, size and temperature of each size range of droplets are determined as functions of time from a set of the above ordinary differential equations and auxiliary algebraic relations subject to initial conditions at t=0. The droplet source terms are obtained by calculating what is lost or gained by the spray in terms of mass, momentum and energy as the droplets enter and leave volume elements of finite dimensions in three dimensional space. It is easy to show, for example, that the mass deposited in an arbitrary control volume by droplets of a given size group is the product of the number flow rate and the difference between droplet mass at entry and at exit. The total mass addition is obtained by summation over all the size groups whose trajectories pass through the control volume in question

$$\dot{m}'' = \sum_{\text{all } i} \{Q_{o,i} [\frac{D^3_{in,i} - D^3_{out,i}}{D^3_{o,i}}]\} \quad \ldots\ldots (30)$$

where Q_o is the mass flow rate of drops of diameter group D_o at the point of spray sheet break-up. The enthalpy and momentum interaction terms are derived by similar reasoning.

Solution Procedure

The previous governing partial differential equations for the conservation of mass, momentum, energy and chemical species for the gaseous phase are reduced into their finite-domain analogues by integration over the computational cells into which the combustor is divided[22]. All the dependent variables, with the exception of velocity components are calculated and stored at the nodal points which these cells encompass. The velocity components lie on the cell boundaries.

The resulting algebraic equations can be represented in the following common form:

$$(\sum_{i=N,S,E,W} A_i - S_p)\,\phi_p = \sum_{i=N,S,E,W} A_i \phi_i + S_U \quad \ldots\ldots (31)$$

where the A's are the coefficients which contain the contributions from the convective and diffusive fluxes, S_U and S_p are the components of the linearized source term.

In the near wall regions, the usual wall functions are matched with the finite-domain equations to preclude fine grid calculations in this region. The usual practice is to cut the link between the boundary and near wall points by setting the appropriate coefficient equal to zero, and to insert the wall influence by way of the linearized source terms[23].

The set of simultaneous algebraic equations can be solved by a semi-implicit iterative scheme which starts from given initial conditions for all the variables and converges to the correct solution on the completion of a number of iterations, provided that the spray-gas interaction terms these equations contain are supplied from the solution of the ordinary differential equations which describe the motion of the droplets.

Each iteration performs the following steps:

i) The u,v and w momentum equations are solved sequentially with guessed pressures.

ii) Since the velocities at this stage do not satisfy the mass continuity equation locally, a "Poisson-type" equation is derived from the continuity equation and the three linearized momentum equations. This pressure-correction equation is then solved for corrections to the pressure field and consequent adjustments are made to the velocity components.

iii) The k and ε equations are solved using the most recent velocity field and the effective viscosity is obtained.

iv) The iteration is completed upon the solution of enthalpy, species conservation and radiation equations.

The sets of simultaneous ordinary differential equations for each of the droplet size ranges are integrated numerically by the Runge-Kutta method at suitable intervals within the above described iterative solution procedure to calculate and up-date the distributions of the spray-gas interaction terms. The gas velocities are assumed, for this purpose, to remain constant within their respective cells. At the end of each Runge-Kutta step, the instantaneous locations of the droplets are checked against the boundaries of the volume elements that they are traversing and when migration into a neighbouring cell occurs, the corresponding values of the gas velocities are substituted into the equations before proceeding into the next time step. This process continues until the droplet disappears by evaporation or impingement on the wall.

Results

A simple gas turbine combustor, on which experimental data is available, was used to validate the mathematical model presented in the preceding sections.

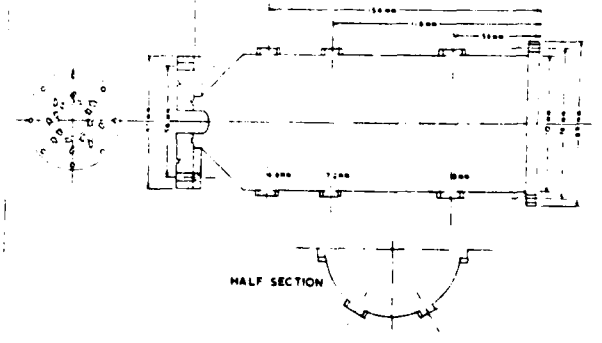

Fig.1. Gas turbine combustor

The geometry of the combustor can, which was designed as a research combustor, is shown in Fig.1. The air stream through the swirler enters the chamber with a finite swirl velocity. Primary, secondary and dilution air streams are introduced via three sets of injection holes, each consisting of six equally spaced circular orifices, giving rise to the three-dimensional nature of the problem. The presumption of cyclic repetition of the flow enables

the combustor can to be represented by a single 60° sector, which is divided into a 27 x 18 x 7 element grid in the x, r and θ directions respectively. This particular combustor does not have any wall cooling slots and it was operated in a large plenum to eliminate any axial components in the radial jet flows. The incoming air flow was divided so that 7.8% of the total flow was admitted through the swirler, and 25.5%, 29.9% and 36.8% through the primary, secondary and dilution jets respectively.

The fuel spray, which is of the hollow cone type with an included angle of 60°, is introduced from an axially located fuel nozzle. The Rosin-Rammler parameters used in equation (18) to represent a typical fuel spray were obtained experimentally[24] using a Malvern particle sizing instrument. From the results typical values of $\bar{D}$ = 60μ and n=2.2 were taken. A total of 20 droplet size ranges and 18 angular injection locations within the 60° sector of the combustor can under consideration are used to construct the complete spray cone. The details of the operating conditions for the combustor are summarized below

Total air flow rate	= 2.125×10^{-2} kg/s
Total fuel flow rate	= 6.25×10^{-4} kg/s
Fuel/Air ratio	= 2.94×10^{-2}
Air temperature at inlet	= 351 K
Swirl number	= 0.8
Initial velocity of droplets	= 20 m/s
Air velocity through injection holes	= 151 m/s
Air velocity through swirler	= 27.6 m/s

A vector plot showing both the direction and the magnitude of velocities in the combustor for the cold, droplet-free flow is given in Fig.2.

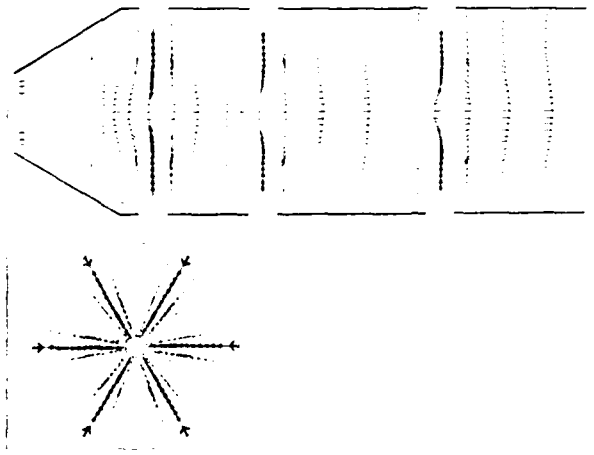

Fig.2. Velocity vectors in the combustor corresponding to the cold, droplet-free flow

It can be noted that air leaving the swirler deflects sharply outwards due to the centrifugal field generated by the inlet swirl and forms a well-defined vortex in the vicinity of the primary jets. All the jets penetrate deep into the flow forming relatively stagnant zones in the wall region between jet inlets. Downstream of the dilution jets, the relatively large radial velocities due to the incoming jets decay substantially and the profile attains a uniform, axial character with a maximum located on the center line.

In Fig.3. the velocity vectors in the combustor for the hot flow are presented. As can be observed most velocities are far in excess of those encountered in the cold case. Hotter regions of the flow domain exhibit high velocity values associated with high temperatures and corresponding low values of density. The recirculation vortex is far stronger in the hot case than in the cold one. It must be mentioned however that in this region of the flow, the spray acts as a constant source of mass and momentum, enhancing greatly the energy content of back flow. The presence of combustion has also

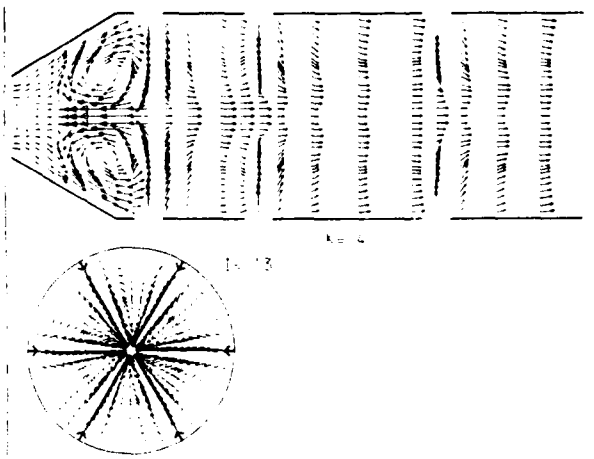

Fig.3. Velocity vectors in the hot reacting flow

helped the velocity field between the primary and secondary jets to gain more uniform and axial description. The exit velocity profiles tend to be more uniform and the secondary and dilution jets do not penetrate as deep into the flow as in the cold case.

The distribution of the mass fraction of fuel evaporated but unburnt is shown in Fig.4. and as expected shows that the majority is found very close to the fuel spray with the effect of recirculation accounting for the fuel-rich region near the axis.

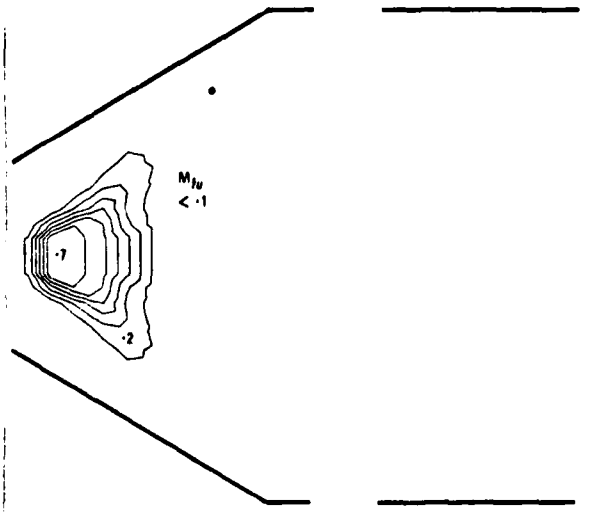

Fig.4. Distribution of mass fraction of unburnt fuel

Knowledge of the distribution of this entity is important for quenching studies, leading for example, to the prediction and minimization of unburned hydrocarbons at engine idle conditions.

The temperature distribution along the combustor for the K=4 plane is displayed in Fig.5. The high temperature region follows the hollow spray

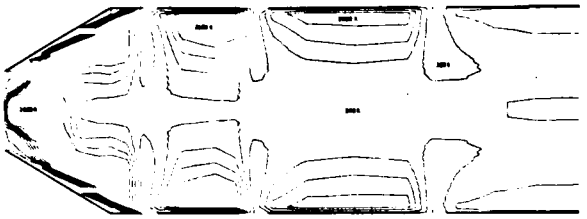

Fig.5. The distribution of isotherms in the combustor

cone with a cooler core caused by the major recirculation eddy. High temperatures also occur in the vicinity of the combustor walls downstream of each air injection position suggesting that combustion may be occurring in these regions. The low temperature primary, secondary and dilution jets can be seen entering the combustor and giving rise to cool core regions. The exit temperature profile which exhibits a minimum on the axis and shows a maximum in the neighbourhood of the walls is consistent with the measurements. Such a profile has been obtained previously by Turan[25] from a pre-mixed pre-vapourized fuel model and therefore cannot be solely attributed to the hollow cone fuel spray. The characteristics of this particular chamber geometry are to blame and the results suggest it should be redesigned with more, smaller air entry ports for practical use in an engine.

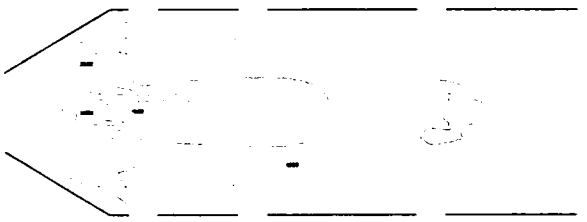

Fig.6. Distribution of the turbulence kinetic energy (m^2/s^2) in the combustor.

Fig.6. shows the distribution of the kinetic energy of turbulence. The most noticeable feature here is the turbulent core near the axis which extends for a large part of the length of the combustion chamber. This large turbulent zone will be of interest for any subsequent study using the chemical reactor network approach.

The droplet trajectories computed during the execution of the two phase flow algorithm can be plotted graphically and are displayed in Fig.7. Here, the spray cone for a single 60^o sector is shown by means of orthographic projection, where the small circles indicate burn-out locations. It can be seen that the droplets deviate from the nominal spray cone by the action of the gas flow quite substantially. Especially, the sudden deflection those droplets which encounter the primary air jet suffer is worth noting. A large number of droplets are found to be hitting the combustor wall, although the majority of the fuel is evaporated near the

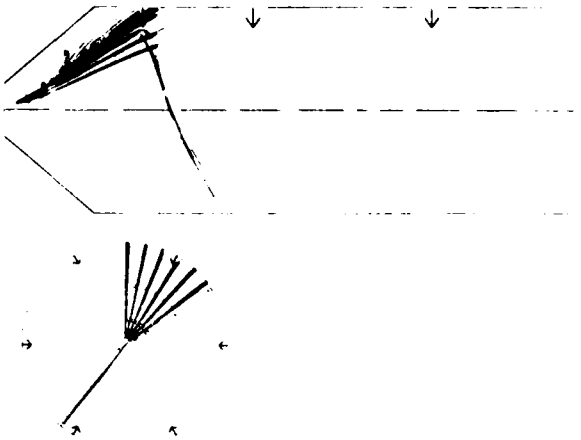

Fig.7. Droplet trajectories for a 60^o sector in orthographic projection

fuel nozzle. For comparison, a 45^o included angle spray injected into the hot flow is depicted in Fig.8. A smaller proportion of the droplets are seen to be hitting the walls of the combustion chamber. Contrary to the behaviour of the previous spray, the droplets are seen to penetrate into the secondary zone. It is apparent from the figure that the three-dimensional flow field has a strong influence on the trajectories of individual droplets.

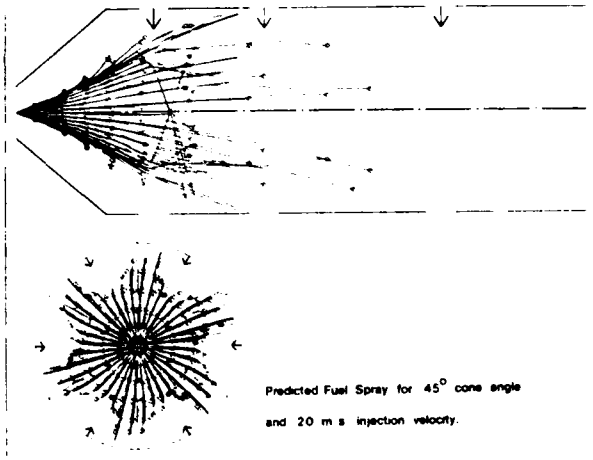

Fig.8. Droplet trajectories for 45^o cone angle.

Conclusions

A mixed Lagrangian-Eulerian two-phase flow prediction algorithm has been formulated and applied to the case of combustion in a gas turbine combustor. The model predicts not only the flow and turbulence field, temperature and species concentration distributions, but also the trajectories of the droplets which constitute the spray.

Despite the complexity of the looping iterative nature of the solution procedure employed in the present study, a better rate and degree of converge than that reported by Turan[25] was achieved.

The computed results reproduced all the pertinent features of the hot reacting flow in gas turbines and were found to be in accord with what

experimental data was available.

It is believed that a stage is now reached where the mathematical modelling of gas turbine combustors presents a practical alternative to empirical methods during the early steps in combustor design. It must be mentioned however, that the generality of the computational procedure remains and it is applicable to any other gas-particle flow problem.

It is endeavoured at present in Sheffield to introduce a more fundamentally based turbulence model into the present algorithm and to account for the turbulent diffusion of particles or droplets by means of a stochastic particle tracking method. Another area of improvement involves the refinement of the droplet evaporation model in the light of the recent developments[26].

It is hoped ultimately to be able to improve the performance of the experimental combustor by means of modifications based on the predictions of the mathematical model.

Acknowledgements

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20. Abstract A Mathematical model of the three-dimensional two-phase reacting flows in gas turbine combustors has been developed which takes into account the mass, momentum and energy coupling between the phases. The fundamental equations of motion of the droplets are solved numerically in a Lagrangian frame of reference using a finite difference solution of the governing equations of the gas. Well known relations are used to model the heat and mass transfer processes and the initial droplet heat-up is allowed for. The entire fuel spray is constructed using a finite number of size ranges obeying a two parameter droplet size distribution. The results are found to be in close agreement with experimental data. An important feature of this analytical technique is that it permits the rational selection or specification of fuel nozzle design.

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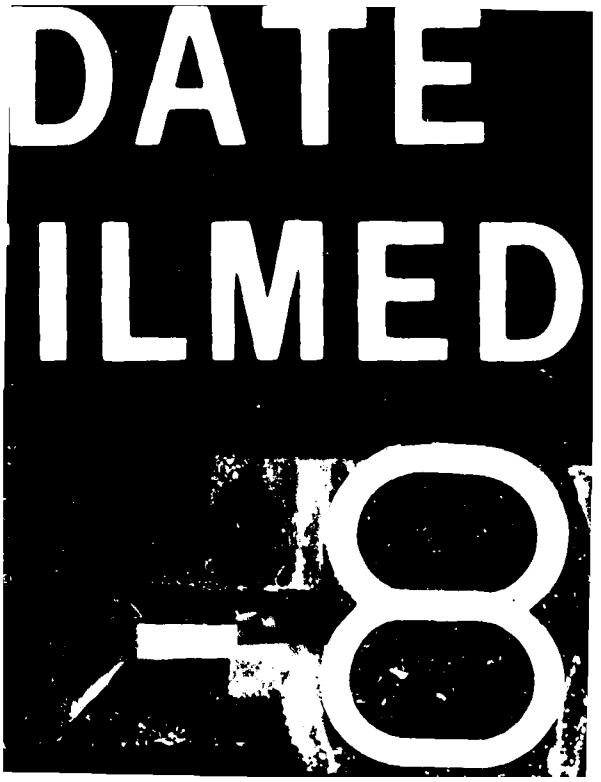
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